Single spin asymmetry in Drell-Yan process

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Outline

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PDFs

- The spin structure of the nucleon can be described through parton distribution functions (PDFs).
- All the structure functions can be expressed by PDFs. For example

$$F_1(x) = \sum_a e_a^2 f(x), \tag{1}$$

  $f(x)$ is the unpolarized PDF.
- Knowing the structure of the nucleon
  $\Rightarrow$ Knowing all the PDFs.
- Different factorizations lead to different PDFs.

TMDs

- Collinear case: only the longitudinal momentum is considered, characterized by a scaling variable x.
- At leading twist, three PDFs are needed, $f(x)$, $g(x)$, $h(x)$.
- TMD case: the transverse momentum of the quarks is taking into account, characterized not only by x, but also by $k_\perp$.
- At leading twist, eight PDFs are needed.
  6 T-even: $f_1(x,k_\perp)$, $g_{1L}(x,k_\perp)$, $g_{1T}(x,k_\perp)$,
  $h_1^\perp(x,k_\perp)$, $h_{1L}^\perp(x,k_\perp)$, $h_{1T}^\perp(x,k_\perp)$,
  2 T-odd: $f_{1T}^\perp(x,k_\perp)$, $h_1^\perp(x,k_\perp)$.
- TMDs give a full 3-D picture of the structure of the nucleon.
- Other framework: Generalized Parton Distributions (GPDs)...

Quark correlator

- The quark-quark correlator (in the light-cone gauge):

$$\Phi(x, p_\perp) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp \cdot \boldsymbol{\xi}_\perp)} \times \langle PS|\bar{\psi}(0)\psi(0,\xi^-,\xi_\perp)|PS\rangle. \quad (2)$$

  Here we omit the gauge link due to the light cone gauge.

- This correlator can be parametrized in a basis of Dirac matrices.

$$\Phi(x, p_\perp) = \frac{1}{2}\Big\{ f_1 \not{n}_+ - f_{1T}^\perp \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} \not{n}_+ + \Big(S_L\, g_{1L} - \frac{p_T \cdot S_T}{M} g_{1T} \Big) \gamma_5 \not{n}_+ + h_{1T} \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} + \Big(S_L\, h_{1L}^\perp - \frac{p_T \cdot S_T}{M} h_{1T}^\perp \Big) \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M} + i\, h_1^\perp \frac{[\not{p}_T, \not{n}_+]}{2M} \Big\}.$$

Definition for TMDs

- Using the trace of the correlator

$$\Phi^{[\Gamma]} \equiv \mathrm{Tr}(\Phi\Gamma)/2 = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp\cdot\boldsymbol{\xi}_\perp)} \times\langle PS|\bar{\psi}(0)\Gamma\psi(0,\xi^-,\xi_\perp)|PS\rangle. \quad (4)$$

We can separate the terms from each other.

- Decomposing the traces of the correlator (for the T-even TMDs only)

$$\Phi^{[\gamma^+]} = f_1, \quad (5)$$

$$\Phi^{[\gamma^+\gamma_5]} = S_\parallel g_{1L} + \frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp}{M_N} g_{1T}, \quad (6)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]} = S^i_\perp h_1 + S_\parallel \frac{p^i_\perp}{M_N} h^\perp_{1L} + S^j_\perp \frac{2p^i_\perp p^j_\perp - p^2_\perp \delta^{ij}_\perp}{2M_N^2} h^\perp_{1T}. \quad (7)$$

Transverse distributions

- We get the transverse TMDs (in Eq. (3)),

$$h_1(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp\cdot\boldsymbol{\xi}_\perp)} \times \langle PS^x|\bar{\psi}(0) i\sigma^{1+}\gamma_5\psi(0,\xi^-,\xi_\perp)|PS^x\rangle, \quad (8)$$

$$\frac{p^x}{M_N} h_{1L}^\perp(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp\cdot\boldsymbol{\xi}_\perp)} \times \langle PS^z|\bar{\psi}(0) i\sigma^{1+}\gamma_5\psi(0,\xi^-,\xi_\perp)|PS^z\rangle, \quad (9)$$

$$\frac{p_\perp^x p_\perp^y}{M_N^2} h_{1T}^\perp(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp\cdot\boldsymbol{\xi}_\perp)} \times \langle PS^y|\bar{\psi}(0) i\sigma^{1+}\gamma_5\psi(0,\xi^-,\xi_\perp)|PS^y\rangle, \quad (10)$$

$|PS^y\rangle$: the hadronic state with a polarization in the y direction.

Transverse distributions

- The hadronic state can be expanded in a series of light-cone Fock states.
- Probability interpretation:
  $h_1(x, p_\perp^2)$: find a transversely polarized quark inside a transversely polarized nucleon carrying the fractional momentum xP and the transverse momentum $p_\perp$;
  $h_{1L}^\perp(x, p_\perp^2)$: find a transversely polarized quark inside a longitudinally polarized nucleon carrying the fractional momentum xP and the transverse momentum $p_\perp$;
  $h_{1T}^\perp(x, p_\perp^2)$: find a transversely polarized quark along x axis inside a transversely polarized nucleon along y axis carrying the fractional momentum xP and the transverse momentum $p_\perp$;

Outline

SU(6) quark-diquark model

- We use the light-cone SU(6) quark-diquark model, in which the proton state is constructed by a valence quark and a spectator diquark.
- The proton state with a spin component $S_z = \pm\frac{1}{2}$ can be written as

$$|p^{\uparrow}\rangle = \frac{1}{3\sqrt{2}}\varphi_V \left[(ud)^0 u^{\uparrow} - \sqrt{2}(ud)^1 u^{\downarrow} - \sqrt{2}(uu)^0 d^{\uparrow} + 2(uu)^1 d^{\downarrow}\right]) + \frac{1}{\sqrt{2}}\varphi_S (ud)^S u^{\uparrow}, \quad (11)$$

$$|p^{\downarrow}\rangle = -\frac{1}{3\sqrt{2}}\varphi_V \left[(ud)^0 u^{\downarrow} - \sqrt{2}(ud)^{-1} u^{\uparrow} - \sqrt{2}(uu)^0 d^{\downarrow} + 2(uu)^{-1} d^{\uparrow}\right] + \frac{1}{\sqrt{2}}\varphi_S (ud)^S u^{\downarrow}, \quad (12)$$

- The $S_x = \pm\frac{1}{2}$ and $S_y = \pm\frac{1}{2}$ can be obtained by

$$|x^{\rightarrow}\rangle = \frac{1}{2}(|p^{\uparrow}\rangle + |p^{\downarrow}\rangle), \; |x^{\leftarrow}\rangle = \frac{1}{2}(|p^{\uparrow}\rangle - |p^{\downarrow}\rangle), \quad (13)$$

$$|y^{\Rightarrow}\rangle = \frac{1}{2}(|p^{\uparrow}\rangle + i|p^{\downarrow}\rangle), \; |y^{\Leftarrow}\rangle = \frac{1}{2}(|p^{\uparrow}\rangle - i|p^{\downarrow}\rangle). \quad (14)$$

Melosh-Wigner rotation

- We need to transform the states from the instant form to the light-front form.
- instant frame $\rightarrow$ light-cone frame: a unitary Melosh-Wigner rotation
  H.J. Melosh, PRD 9, 1095 (1974); E. Wigner, AM 40, 149 (1939). B.-Q. Ma, JPG 17, L53 (1991).
- For a spin-$\frac{1}{2}$ particle

$$\begin{pmatrix} q_F^{\uparrow} \\ q_F^{\downarrow} \end{pmatrix} = \omega \begin{pmatrix} k^+ + m & -k^R \\ k^L & k^+ + m \end{pmatrix} \begin{pmatrix} q_T^{\uparrow} \\ q_T^{\downarrow} \end{pmatrix} \equiv \mathbf{M}^{1/2} \begin{pmatrix} q_T^{\uparrow} \\ q_T^{\downarrow} \end{pmatrix},$$

- Convert the instant wave functions to the light cone wave functions.

TMDs in light-cone SU(6) quark-diquark model

- Using the two-particle Fock state expansion with the light cone wave functions, We could calculate the trace of the correlator in our model.

$$\begin{aligned}\phi^{[\Gamma]} &= \sum_{j,\lambda,\lambda',\lambda_D} \frac{1}{32\pi^3}\frac{1}{xP^+} \\ &\times \psi_j^*(x,\mathbf{p}_\perp,\lambda;1-x,-\mathbf{p}_\perp,\lambda_D)\psi_j(x,\mathbf{p}_\perp,\lambda';1-x,-\mathbf{p}_\perp,\lambda_D) \\ &\times \bar{u}(xP^+,\mathbf{p}_\perp,\lambda)\Gamma u(xP^+,\mathbf{p}_\perp,\lambda'). \end{aligned} \tag{15}$$

- Results in our model

$$\begin{aligned} f_1^{(uv)}(x,\mathbf{p}_\perp) &= \frac{1}{32\pi^3}\times(\frac{1}{3}\varphi_V^2+\varphi_S^2), \\ f_1^{(dv)}(x,\mathbf{p}_\perp) &= \frac{1}{16\pi^3}\times\frac{1}{3}\varphi_V^2; \end{aligned} \tag{16}$$

$$\begin{aligned} h^{(uv)}(x,\mathbf{p}_\perp) &= -\frac{1}{32\pi^3}\times(\frac{1}{9}\varphi_V^2 W_V-\varphi_S^2 W_S), \\ h^{(dv)}(x,\mathbf{p}_\perp) &= -\frac{1}{16\pi^3}\times\frac{1}{9}\varphi_V^2 W_V, \end{aligned} \tag{17}$$

TMDs in light-cone SU(6) quark-diquark model

- h is denoted for the transverse TMDs, i.e., h_1, $h_{1L}^{\perp}$, $h_{1T}^{\perp}$.
- $\varphi_V(\varphi_S)$: the wave functions in the momentum space.
- $W_D(D = V, S)$: the Melosh-Wigner rotation factor.
- All the polarized TMDs have the same form for the expression, but with different Melosh-Wigner rotation factors.

$$\text{For transversity}: W_D(x, \mathbf{p}_\perp) = \frac{(x\mathcal{M}_D + m_q)^2}{(x\mathcal{M}_D + m_q)^2 + p_\perp^2} \quad (18)$$

$$\text{For longi} - \text{transversity}: W_D(x, \mathbf{p}_\perp) = -\frac{2M_N(x\mathcal{M}_D + m_q)}{(x\mathcal{M}_D + m_q)^2 + p_\perp^2} \quad (19)$$

$$\text{For pretzelosity}: W_D(x, \mathbf{p}_\perp) = -\frac{2M_N^2}{(x\mathcal{M}_D + m_q)^2 + p_\perp^2} \quad (20)$$

with $\mathcal{M}_D^2 = \frac{m_q^2+p_\perp^2}{x} + \frac{m_D^2+p_\perp^2}{1-x}$

- Relation between polarized and unpolarized distributions.

$$\begin{aligned} h^{(uv)}(x,\mathbf{p}_\perp) &= \left[f_1^{(uv)}(x,\mathbf{p}_\perp) - \frac{1}{2} f_1^{(dv)}(x,\mathbf{p}_\perp) \right] W_S(x,\mathbf{p}_\perp) \\ &\quad -\frac{1}{6} f_1^{(dv)}(x,\mathbf{p}_\perp) W_V(x,\mathbf{p}_\perp), \\ h^{(dv)}(x,\mathbf{p}_\perp) &= -\frac{1}{3} f_1^{(dv)}(x,\mathbf{p}_\perp) W_V(x,\mathbf{p}_\perp). \end{aligned} \tag{21}$$

- It is approximately satisfied. It is valid at an initial scale and the evolution effect for the polarized distribution is partially contained in the unpolarized distribution.
- It will be used to calculate the polarized distribution. We can adopt a parametrization for f_1 as an input, since we know more about f_1 than the light-cone wave function. The only change is the Melosh-Wigner rotation factors.

Outline

Drell-Yan process

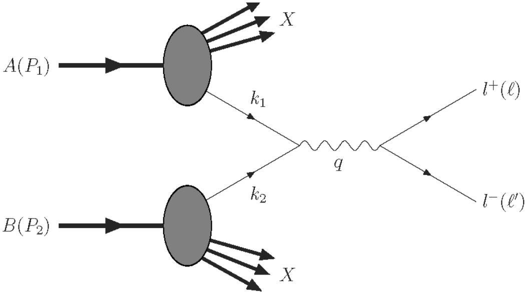

Figure: Drell-Yan dilepton process.

- At COMPASS, $\pi p \rightarrow \mu^+\mu^- X$, where p is longitudinal or transversely polarized.

Kinematical variables

- Then the momentum transfer gives the invariant mass of the lepton pair

$$q^2 = (k_1 + k_2)^2 = (\ell^+ + \ell^-)^2 = M^2. \tag{22}$$

- At extremely high energies, all the mass effects and the transverse momenta are ignored,

$$x_1 = \frac{q^2}{2P_1 \cdot q} \approx \frac{q_0 + q_L}{\sqrt{s}}, \qquad x_2 = \frac{q^2}{2P_2 \cdot q} \approx \frac{q_0 - q_L}{\sqrt{s}},$$
$$\tau = \frac{M^2}{s}, \qquad x_F = x_1 - x_2 \approx \frac{2q_L}{\sqrt{s}}. \tag{23}$$

- We can build up the relation

$$x_1 = \frac{1}{2}\Big(x_F + \sqrt{x_F^2 + 4\tau}\Big),$$
$$x_2 = \frac{1}{2}\Big(-x_F + \sqrt{x_F^2 + 4\tau}\Big). \tag{24}$$

Cross section and SSA

- The cross section is

$$
\begin{aligned}
&\frac{d\sigma}{d\Omega dx_1 dx_2 d^2\mathbf{q}_T} = \frac{\alpha^2}{3q^2}\{A(y)\mathcal{F}[\bar{f}_1 f_1] \\
&+S_{2L}B(y)\sin(2\phi)\times\mathcal{F}[\frac{2(\hat{\mathbf{h}}\cdot\mathbf{p}_{1T})(\hat{\mathbf{h}}\cdot\mathbf{p}_{2T})-\mathbf{p}_{1T}\cdot\mathbf{p}_{2T}}{M_1M_2}\bar{h}_1^\perp h_{1L}^\perp]+\cdots, \\
&-|\mathbf{S}_{2T}|B(y)[\sin(\phi+\phi_{S_2})\times\mathcal{F}[\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_{1T}}{M_1}\bar{h}_1^\perp h_1]+\sin(3\phi-\phi_{S_2}) \\
&\times\mathcal{F}[\frac{4\hat{\mathbf{h}}\cdot\mathbf{p}_{1T}(\hat{\mathbf{h}}\cdot\mathbf{p}_{2T})^2-2\hat{\mathbf{h}}\cdot\mathbf{p}_{2T}\mathbf{p}_{1T}\cdot\mathbf{p}_{2T}-\hat{\mathbf{h}}\cdot\mathbf{p}_{1T}\mathbf{p}_{2T}^2}{2M_1M_2^2}\bar{h}_1^\perp h_{1T}^\perp]]\} \\
&+\ldots, \\
&=\frac{\alpha^2}{3q^2}\{A(y)F_{UU}+|\mathbf{S}_{2L}|B(y)\sin(2\phi)F_{UL}^{\sin(2\phi)} \\
&+|\mathbf{S}_{2T}|B(y)[\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)}+\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}]\} \\
&+\ldots
\end{aligned}
\tag{25}
$$

Cross section and SSA

- We have used the shorthand

$$
\begin{aligned}
A(y) &= \frac{1}{2} - y + y^2 \overset{\text{cm}}{=} \frac{1}{4}(1+\cos^2\theta), \\
B(y) &= y(1-y) \overset{\text{cm}}{=} \frac{1}{4}\sin^2\theta. \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}[\omega \bar{f} g] &\equiv \sum_{a,\bar{a}} \int d\mathbf{p}_{1T} d\mathbf{p}_{2T} \delta^2(\mathbf{p}_{1T} + \mathbf{p}_{2T} - \mathbf{q}_T)\omega(\mathbf{p}_{1T}, \mathbf{p}_{2T}) \\
&\times \bar{f}^{\bar{a}}(x_1, \mathbf{p}_{1T}) g^a(x_2, \mathbf{p}_{2T}) \qquad (27)
\end{aligned}
$$

- The single spin asymmetry is defined as

$$
A = \frac{1}{|S|} \frac{d\sigma(\vec{S}) - d\sigma(-\vec{S})}{d\sigma(\vec{S}) + d\sigma(-\vec{S})} \qquad (28)
$$

Weighted SSA

- In practice, we multiply different weighting functions to separate different azimuthal angle dependence asymmetry.

$$A_{UL(T)}^{W(\phi,\phi_S)} = \frac{2\int_0^{2\pi} d\phi W(\phi,\phi_S)[d\sigma^{\Rightarrow(\uparrow)} - d\sigma^{\Leftarrow(\downarrow)}]}{|S_{L(T)}|\int_0^{2\pi} d\phi[d\sigma^{\Rightarrow(\uparrow)} + d\sigma^{\Leftarrow(\downarrow)}]} \propto \frac{F_{UL(T)}^{W(\phi,\phi_S)}}{F_{UU}}. \quad (29)$$

- We will calculate the following asymmetries by

$$A_{UL}^{\sin(2\phi)} = \frac{B(y)F_{UL}^{\sin(2\phi)}}{A(y)F_{UU}}, \quad (30)$$

$$A_{UT}^{\sin(\phi+\phi_S)} = \frac{B(y)F_{UT}^{\sin(\phi+\phi_S)}}{A(y)F_{UU}}, \quad (31)$$

$$A_{UT}^{\sin(3\phi-\phi_S)} = \frac{B(y)F_{UT}^{\sin(3\phi-\phi_S)}}{A(y)F_{UU}}. \quad (32)$$

Parametrization

- To make our result more reliable, we adopt CETQ parametrization for $f_1(x)$ as an input. The transverse momentum dependence is the Gaussian form for the unpolarized distribution.
- Using the relation (33) to obtain the polarized PDFs,

$$\begin{aligned} h^{(uv)}(x, \mathbf{p}_\perp) &= \left[f_1^{(uv)}(x, \mathbf{p}_\perp) - \frac{1}{2} f_1^{(dv)}(x, \mathbf{p}_\perp) \right] W_S(x, \mathbf{p}_\perp) \\ &\quad - \frac{1}{6} f_1^{(dv)}(x, \mathbf{p}_\perp) W_V(x, \mathbf{p}_\perp), \\ h^{(dv)}(x, \mathbf{p}_\perp) &= -\frac{1}{3} f_1^{(dv)}(x, \mathbf{p}_\perp) W_V(x, \mathbf{p}_\perp). \end{aligned} \quad (33)$$

- We need the Boer-Mulders functions $h_1^\perp$ for pion.
Z. Lu, B.-Q. Ma, PLB 615 (2005) 200;

Kinematics

- The COMPASS kinematics,

$$\sqrt{s} = 18.9\ \mathrm{GeV},\ 0.1 < x_1 < 1,\ 0.05 < x_2 < 0.5,$$
$$4 \leqslant M \leqslant 8.5\ \mathrm{GeV},\ 0 \leqslant q_T \leqslant 4\ \mathrm{GeV}\ (\text{if } q_T \text{ is integrated}).$$

- For the x_F dependence, we only give the prediction for the forward region $x_F > 0$. Given a fixed x_F^0, the range for M is determined by Eq. (24) so that $x_{1,2}^{\min} < x_{1,2}(x_F^0, M) < x_{1,2}^{\max}$.
- For the M dependence, given a fixed M_0, the range for x_F is determined by Eq. (24) so that $x_{1,2}^{\min} < x_{1,2}(x_F, M_0) < x_{1,2}^{\max}$.
- For the q_T dependence, the range for M is $4 \leqslant M \leqslant 8.5\ \mathrm{GeV}$, and the range for x_F is determined by Eq. (24) so that $x_{1,2}^{\min} < x_{1,2}(x_F, M) < x_{1,2}^{\max}$.

$\sin 2\phi$ asymmetry

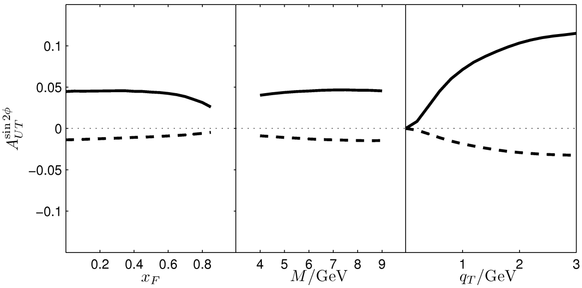

Figure: The $\sin 2\phi$ asymmetries for $\pi^{\pm} p^{\Rightarrow} \rightarrow \mu^{+}\mu^{-}X$ process at COMPASS. Solid and dashed curves are the results for π^{-} and π^{+} beams, respectively.

$\sin(\phi + \phi_S)$ asymmetry

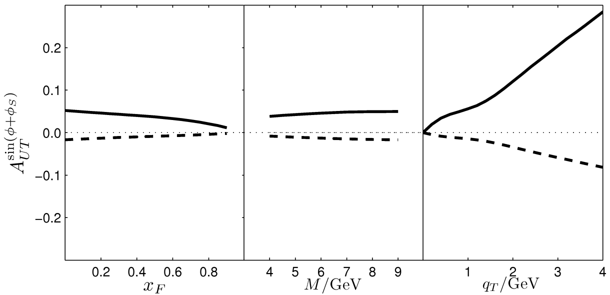

Figure: The $\sin(\phi + \phi_S)$ asymmetries for $\pi^{\pm} p^{\uparrow} \rightarrow \mu^{+}\mu^{-}X$ process at COMPASS. Solid and dashed curves are the results for π^{-} and π^{+} beams, respectively.

$\sin(3\phi - \phi_S)$ asymmetry

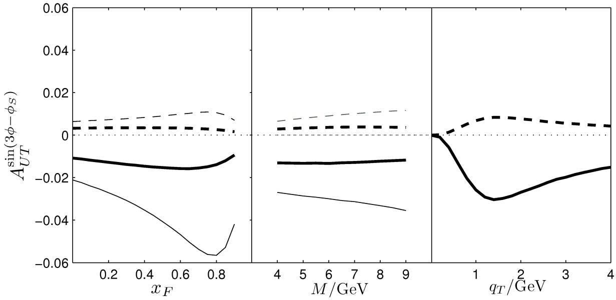

Figure: The $\sin(3\phi - \phi_S)$ asymmetries for $\pi^{\pm} p^{\uparrow} \to \mu^+ \mu^- X$ process at COMPASS. Solid and dashed curves are the results for π^- and π^+ beams, respectively. Thin curves are the results where we make a cut off on q_T to enhance the asymmetry.

Comment

- All the results above rely on the Boer-Mulders functions in pion, which is little known yet.
- The $\sin 2\phi$ and $\sin(\phi + \phi_S)$ asymmetries are a few percent, thus can be measured with a good accuracy.
- The $\sin(3\phi - \phi_S)$ asymmetry is a little small, less than 1%, thus bring in difficulty in measuring it (Thick curves in Fig. 4).
- In order to enhance the asymmetry, we make a cut-off on q_T, $1.0 \leqslant q_T \leqslant 2.0$ GeV. The asymmetry could be magnified to a few percent (Thin curves in Fig. 4), although we may suffer a loss of data.
- Results in large q_T region ($q_T > 3$ GeV) might be not so reliable.

Outline

Summary

- We present our model calculation on $h_{1L}^{\perp},\ h_1,\ h_{1T}^{\perp}$.
- They can be probed through $\sin 2\phi$ asymmetry with longitudinal polarized proton, $\sin(\phi+\phi_S)$ asymmetry with transversely polarized proton and $\sin(3\phi-\phi_S)$ asymmetry with transversely polarized proton in πp Drell-Yan process.
- Boer-Mulders function for pion is needed.
- The $\sin 2\phi$ and $\sin(\phi+\phi_S)$ asymmetry are a few percent.
- The $\sin(3\phi-\phi_S)$ asymmetry is a little small. We could make a cut-off on q_T to enhance the asymmetry.
- We expect future experiments could promote our understanding on the nucleon spin structure.

Thank you!